OPTIMIZATION OF THERMODIFFUSION COLUMN UNDER EXTRACTION CONDITIONS

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It is shown that under extraction conditions there exists an optimal size of a gap for which the column productivity is the highest.

It is known that in an ideal cascade [1] the lowest energy consumption can be obtained for the separation process of a binary mixture. In such a cascade the dimensionless extraction value is one half of the maximally possible, the latter being given by

$$\varkappa = \frac{c_i(1-c_i)}{c_e - c_i}, \tag{1}$$

and the specific heat consumption per unit of the enriched product when a constant concentration is maintained at one end of the cascade is equal to the original one [2, 3]:

$$Q_{\text{sp}}^{\text{id}} = \frac{40}{7}\cdot\frac{\lambda}{\alpha^2 \rho D}\cdot\frac{\bar{T}^2}{\Delta T} V(c_e, c_0). \tag{2}$$

Practical implementation of an ideal cascade profile is not possible; it can be approximated, however, by a stepwise cascade with lower energy efficiency.

It is shown below that in some cases for a specified choice of geometric characteristics and of operating state parameters of a column of constant cross section its energetics efficiency may not be smaller than that of a stepwise cascade.

To this end we shall consider the case which is of interest in practice of removing a small amount of admixture from the main substance with the initial concentration c_0, that is, the case of $1 - c \ll 1$.

Then in accordance with [4] the separation degree is related to the parameters $\varkappa$ and y_e by the relation

$$q(1-\varkappa) + \varkappa = e^{y_e(1-\varkappa)}. \tag{3}$$

The optimization problem consists in finding for given separation degree and temperature states of the column operation those geometric characteristics for which its performance will be the highest possible.

The quantities y_e and $\varkappa$ which appear in (3) are functions of the gap between the hot and the cold surfaces of the column, that is,

$$y_e = \frac{y_e^*}{\delta^4}, \quad \varkappa = \frac{\sigma}{H^* \delta^3}. \tag{4}$$

In accordance with what was stated above, the extraction magnitude is also regarded by us as a function of the gap, that is, $\sigma = \sigma(\delta)$.

By differentiating (3) with respect to δ and setting $d\sigma/d\delta = 0$, the following expression for the optimal parameter is obtained:

$$y_{e,\text{opt}} = \frac{3\varkappa_{\text{opt}}(q-1)}{(7\varkappa_{\text{opt}} - 4)\left[\varkappa_{\text{opt}} + q(1-\varkappa_{\text{opt}})\right]}. \tag{5}$$

If one inserts (5) into (3), one obtains the following transcendental equation for the optimal value of $\varkappa$:

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TABLE 1. Optimal Values of Dimensionless Lengths and Extractions Depending on Separation Degree in Accordance with (5) and (6)

q	1,5	5	10	20	50	100	150
$\varkappa_{\text{opt}}$	2,287	0,8950	0,7704	0,7085	0,6655	0,64663	0,6385
$y_{e,\,\text{opt}}$	0,804	3,37	4,88	6,39	8,54	10,13	11,08

$$q(1-\varkappa_{\text{opt}})+\varkappa_{\text{opt}}=\exp\left\{\frac{3\varkappa_{\text{opt}}(1-\varkappa_{\text{opt}})(q-1)}{(7\varkappa_{\text{opt}}-4)[\varkappa_{\text{opt}}+q(1-\varkappa_{\text{opt}})]}\right\}. \tag{6}$$

It can be seen from the formulas (5) and (6) that the optimal values of y_e and $\varkappa$ only depend on the separation degree. Corresponding numerical values are shown in Table 1.

By using (4) one can determine for specified column length and temperature states the optimal gap magnitude:

$$\delta_{\text{opt}}=\left(\frac{y_e^*}{y_{e,\text{opt}}}\right)^{1/4}, \tag{7}$$

where $y_{e,\text{opt}}$ is given by (5).

The quantity y_e^* is not completely arbitrary, since it is related to the constraints imposed by the hydrodynamic conditions in the column as well as by the separation conditions of the mixture components.

From the latter point of view and with a fixed length and specified separation degree the maximal gap corresponds to no extraction from the column ($\varkappa = 0$); this, according to (3), yields $y_e = \ln q$, and in view of the first relation of (4) determines an upper bound for the values of y_e^* by means of the inequality

$$y_e^* \leqslant \delta_{\max}^4 \ln q. \tag{8}$$

On the other hand, the condition of hydrodynamic stability according to which the velocity profile corresponding to a cubic parabola is maintained along the entire height of the separation aperture demands that the Rayleigh number be within the limits 10^3-10^5 as shown experimentally in [5]. For the sake of reliability let us set

$$\mathrm{Ra} \equiv \frac{g\rho\beta(\Delta T)\delta_*^3}{a\eta} \leqslant 10^3. \tag{9}$$

Since $\delta_{\max} \leq \delta_*$, from (8) one obtains by employing (9)

$$y_e^* \leqslant 10^4\left(\frac{a\eta}{g\rho\beta\Delta T}\right)^{\frac{4}{3}} \ln q, \tag{10}$$

and hence the maximum feasible height of the column can be obtained:

$$L_{\max} \leqslant 20\left(\frac{\eta}{\rho g\beta}\right)^{1/3}\left(\frac{a}{\Delta T}\right)^{4/3}\frac{\overline{T}}{\alpha D}\ln q, \tag{11}$$

for which the required hydrodynamic conditions are maintained.

It may turn out that the parameter y_e^* selected in accordance with (10) is such that δ_{opt} as given by (7) will be too small, that is, it could not be technically realizable. It is then advisable to subdivide the entire column into n equal stages which are joined together by thermosiphon loops.

For each of these n stages the optimal value of y_e is the n-th part of $y_{e,\text{opt}}$ as given by (5); (7) is now replaced by

$$\delta_{\text{opt}}=\left(\frac{ny_e^*}{y_{e,\text{opt}}}\right)^{1/4}. \tag{12}$$

If the column performance is given, then the second relation of (4) enables one to determine

$$H^*=\frac{\sigma_{\text{opt}}}{\varkappa_{\text{opt}}\delta_{\text{opt}}^3}, \tag{13}$$

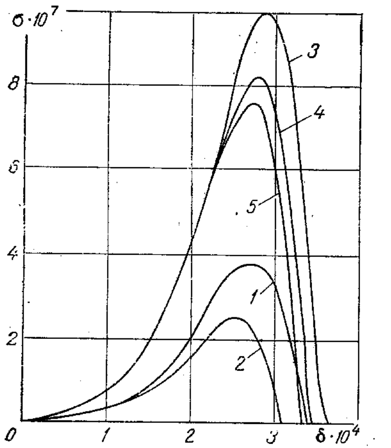

Fig. 1

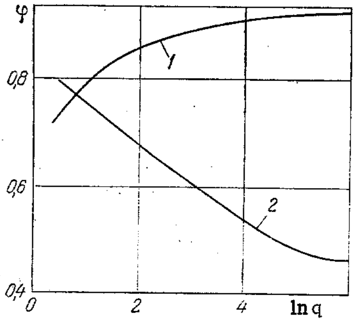

Fig. 2.

Fig. 1. Column productivity versus slot size δ: $H^* = 2 \cdot 10^4$ kg/m^3·sec, $y_e^* = 2 \cdot 10^{-14}$ m^4: 1) $q = 5$, 2) 10; $H^* = 2 \cdot 10^5$ kg/m^3·sec, $y_e^* = 6 \cdot 10^{-14}$ m^4: 3) 50; 4) 100; 5) 150; $\sigma \cdot 10^7$ kg·sec^{-1}; $\delta \cdot 10^4$, m.

Fig. 2. Relative efficiency of an optimized thermodiffusion column of constant section compared with an ideal cascade versus separation degree: 1) $1 - c \ll 1$; 2) $c \ll 1$ (the values of ln q are shown on the abscissa axis).

and, consequently, the required width of the separating aperture B. If, on the other hand, the quantity H^* is specified, then (13) can be used to find the maximal column performance. In Fig. 1 the computation results are shown by employing the above formulas for various values of H^* and y_e^*. It can be seen from the diagram that in the region of $\delta > \delta_{opt}$ the column performance worsens rapidly for wider gaps and that this region should be regarded as the instability region for the column operation, since small swings in the value of H^* due to variations of ΔT caused by the instability of heat-exchange conditions on the thermostatically controlled column surfaces can have a considerable effect on the amount of extraction or, with the latter remaining constant, on the attainable separation degree.

It can also be seen from the diagram that for lower degrees of separation the maxima of the curves become more flat and there is less need to observe strictly the nominal value of the optimal gap.

A comparison is now carried out of the energetic efficiency of the optimized thermodiffusion column of constant section and the ideal cascade, both with the same gap values and with the same width of the separating aperture in the null section of the ideal cascade and in the optimized column.

The heat consumption in the column is given by

$$Q = F \frac{\lambda}{\delta_{opt}} \Delta T. \tag{14}$$

The surface area of the column is $F = BL$ and its height can be expressed as $L = y_{e,opt} K_{opt}/H_{opt}$, which replaces (14) by

$$Q = y_{e,\,opt} \frac{K_{opt} B}{H_{opt}} \cdot \frac{\lambda}{\delta_{opt}} \Delta T.$$

Dividing both sides of the above expression by σ and using the second relation of (4), one obtains an expression for the specific heat consumption in the optimized thermodiffusion column of constant section, namely,

$$Q_{sp} = \frac{y_{e,\,opt}}{\varkappa_{opt}} \cdot \frac{K_{opt} B}{H_{opt}^2} \cdot \frac{\lambda}{\delta_{opt}} \Delta T. \tag{15}$$

By replacing $y_{e,opt}$ by its value in (5) and K_{opt} and H_{opt} by their respective expressions (see "Notation"), one finally obtains

$$Q_{\mathrm{sp}} = \frac{30}{7} \frac{(q-1)}{(7\varkappa_{\mathrm{opt}}-4)[\varkappa_{\mathrm{opt}}+q(1-\varkappa_{\mathrm{opt}})]} \cdot \frac{\lambda}{\alpha^2 \rho D} \cdot \frac{\bar{T}^2}{\Delta T} \cdot \tag{16}$$

The value function which appears in (2) is given by

$$V(e_e, c_0) = \frac{(c_e - c_0)(1-2c_0)}{c_0(1-c_0)} - (1-2c_e)\ln q \tag{16a}$$

which in the case under consideration (1 − c ≪ 1) becomes

$$V(c_e, c_0) = \ln q + \frac{1}{q} - 1. \tag{17}$$

By replacing $V(c_e, c_0)$ in (2) in accordance with (17) and dividing by (16), one obtains

$$\varphi = \frac{Q_{\mathrm{sp}}^{\mathrm{id}}}{Q_{\mathrm{sp}}} = \frac{4}{3} \cdot \frac{q\ln q - q + 1}{q(q-1)} (7\varkappa_{\mathrm{opt}}-4)[\varkappa_{\mathrm{opt}}+q(1-\varkappa_{\mathrm{opt}})]. \tag{18}$$

It can be seen that the **ratio which** is of interest to us is a function of the separation degree only. The function is shown in Fig. 2; it shows that φ approaches unity if the separation degree increases, that is, the efficiency of the optimized column of constant section differs only slightly from that of an ideal cascade and is not inferior to that of a stepwise cascade.

This result, which is expressed by the formula (18), is not, however, general; that is, it cannot be extended to any concentrations of the original mixture.

If the second limiting case is considered, namely, that of low concentration of the end component, that is, c ≪ 1, then in accordance with [4] instead of (3) one has

$$\frac{1}{q}(1+\varkappa) - \varkappa = e^{-y_e(1+\varkappa)} . \tag{19}$$

By repeating our previous considerations, instead of the formulas (5), (6), and (18), we obtain, respectively,

$$y_{e,\mathrm{opt}} = \frac{3\varkappa_{\mathrm{opt}}(q-1)}{(7\varkappa_{\mathrm{opt}}+4)[1-\varkappa_{\mathrm{opt}}(q-1)]}, \tag{20}$$

$$\frac{1}{q}(1+\varkappa_{\mathrm{opt}}) - \varkappa_{\mathrm{opt}} = \exp\left\{-\frac{3\varkappa_{\mathrm{opt}}(1+\varkappa_{\mathrm{opt}})(q-1)}{(7\varkappa_{\mathrm{opt}}+4)[1-\varkappa_{\mathrm{opt}}(q-1)]}\right\}, \tag{21}$$

$$\varphi = \frac{4}{3} \cdot \frac{q-1-\ln q}{q-1} (7\varkappa_{\mathrm{opt}}+4)[1-\varkappa_{\mathrm{opt}}(q-1)]. \tag{22}$$

The relation (22) is also shown in Fig. 2; it shows an increase in the efficiency of the optimized column with the separation degree decreasing, that is, the least interesting case in practice.

It has been shown by us that if thermodiffusion is used for matter removal, there is no need to apply a stepwise cascade of variable profile, since the optimized column of constant section proves energetically equivalent to it.

Nevertheless, it should be mentioned here that this conclusion is valid if the duration of the transient process is brief from the point of view of economical engineering indices.

The above considerations are valid for an ideal thermodiffusion column, that is, for a column such that both thermostatically controlled surfaces are ideally isothermic. In a real column there is always temperature asymmetry which results in a parasitic convection whose existence even under no extraction conditions of the column shifts the optimal value of the operating gap to higher values [8]. Therefore, the results as given by the formula (5) are underestimates and should be improved by taking into account the effect of parasitic convection; the latter should be investigated further.

In conclusion, it is noted that optimization of a thermodiffusion column was also considered in [6, 7] though from a different point of view. In [6] it was shown that there is an optimal flow rate of heat so that the separation degree is the highest possible; this result was obtained by taking into account that the coefficients in the transfer equation depend on the amount of extraction. This dependence is very weak, however, and is usually ignored; thus, the conclusions of [6] are of no practical value.

As regards [7], the slope angle is optimized of a flat thermodiffusion column which ensures the highest separation degree.

NOTATION

c, concentration; λ, D, η, β, coefficients of thermal conductivity, diffusion, dynamic viscosity, and volume expansion; ρ, density; α, thermodiffusion constant; $\bar{T}$, mean temperature in operational gap; ΔT, temperature difference; $\varkappa = \sigma/H$; σ, column performance (extraction); $V(c_e, c_0)$, value function, see (16a); $q = c_e(1 - c_0)/c_0(1 - c_e)$; $H = \alpha g \rho^2 \beta \delta^3 (\Delta T)^2 B/6!\eta\bar{T}$, H^*, see (4); $y_e = 504\alpha\eta D L/\rho g \beta \delta^4 \bar{T}$, y_e^*, see (4); δ, gap; L, column length; a, thermal diffusivity. Indices: 0, initial state; e, value near the refining end of column.

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DYNAMICS OF PARTICLES IN A SPIRAL FLOW

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Dynamic characteristics have been derived for particles falling under gravity in a spiral heat-carrier flow.

Many theoretical studies have been made on the motion of small particles in turbulent flows, of which Chen's papers [1] form an important part.

Here we use Chen's equations [1] to determine the dynamic characteristics of a particle falling under gravity in a spiral flow within a rotating drying drum. The calculations are compared with experiment for particles falling in a homogeneous flow.

The equation of motion for a small spherical particle falling under gravity in a turbulent spiral gas flow within a rotating cylindrical drying drum takes the form

$$\frac{\pi}{6} d^3 \rho \vec{w} = \frac{\pi}{6} d^3 \rho \vec{g} + \frac{c_x}{2} \rho_1 \frac{\pi d^2}{4} |\vec{v}_1 - \vec{v}| (\vec{v}_1 - \vec{v}). \tag{1}$$

Equation (1) is derived from Chen's equation [1] by neglecting forces related to the acceleration of the fluid, as well as those due to the adjoint mass and the Basset force, which incorporates the history of the particle acceleration; this is done because these forces are small if the fluid is of low speed (3-7 m/sec) and the circumferential velocity is small ($v_\varphi = 1$ m/sec). Also, an assumption different from Chen's is that the viscous resistance is proportional to the square of the relative velocity $(\vec{v}_1 - \vec{v})$ [2], with this force directed against the flow direction if the particle moves faster than the liquid, and vice versa.

We solve (1) for $\vec{w}$ on the basis that $c_x = 0.5$ [2], which gives

$$\vec{w} = \vec{g} + \frac{3}{8} \frac{\rho_1}{\rho} \frac{1}{d} |\vec{v}_1 - \vec{v}| (\vec{v}_1 - \vec{v}). \tag{2}$$

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